Why Are Things Shaped the Way They Are?

Why do corn kernels grow in staggered rows? Why are manhole covers always round and honeycomb cells hexagonal? Important geometric concepts are embedded in the shape and design of natural and manufactured objects.

Why are manhole covers round? They are so much a part of our daily landscape that we hardly notice them. However, the fact that they are circles is no accident (NCTM 2004). This article examines how children can explore important geometric concepts that are embedded in the shape and design of both natural and manufactured objects. Posing and investigating the question of why objects are shaped the way they are is consistent with NCTM's *Principles and Standards for School Mathematics* (2000), which advocates the application of geometry in real-world situations. Such a question can be a valuable central thread in the geometry curriculum of the elementary grades. For example, looking closely at the everyday manhole cover can help children identify unique attributes of a circle.

One fourth-grade class investigated the manhole cover question by experimenting with models. As part of their mathematics curriculum, the children had to be able to identify the attributes of regular polygons. The following activity afforded them the opportunity to compare these shapes in terms of sides, angles, number of diagonals, and symmetry. The teacher also chose this activity because it placed learning these skills into a real-world context. She prepared materials for the lesson by using a utility knife to cut convex shapes from card stock, leaving the resulting hole intact as a frame. Each group of children received a set of shapes consisting of a circle and nine polygons: four quadrilaterals (a trapezoid, a rectangle, a square, and a rhombus), two triangles (an isosceles and an equilateral), an octagon, a hexagon, and a regular pentagon (see **fig. 1**). Students were instructed to record whether or not each shape could pass through its frame without bending and without touching the frame, to describe the differences among the shapes, and to defend why manhole covers are circular.

The children identified the circle as the only shape that could not pass through its own frame. All the polygons that the children tried could be rotated so that a side passed through one of the frame's diagonals. The children observed that the circle's center is equidistant from all its edges. Because all diameters

By David J. Whitin and Phyllis Whitin


David J. Whitin, david.whitin@wayne.edu, teaches mathematics and general education courses, and Phyllis Whitin, phyllis.whitin@wayne.edu, teaches language arts and children's literature. They both teach at Wayne State University in Detroit, Michigan, and work regularly with teachers in their classrooms to build responsive mathematical communities.

Figure 1

Students received a set of ten shapes—a circle and nine polygons—for the Manhole Cover investigation.

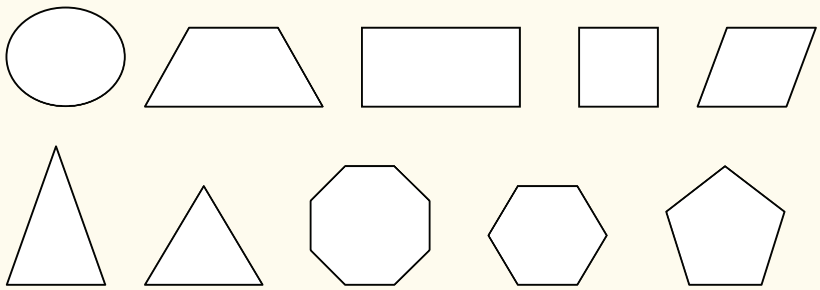
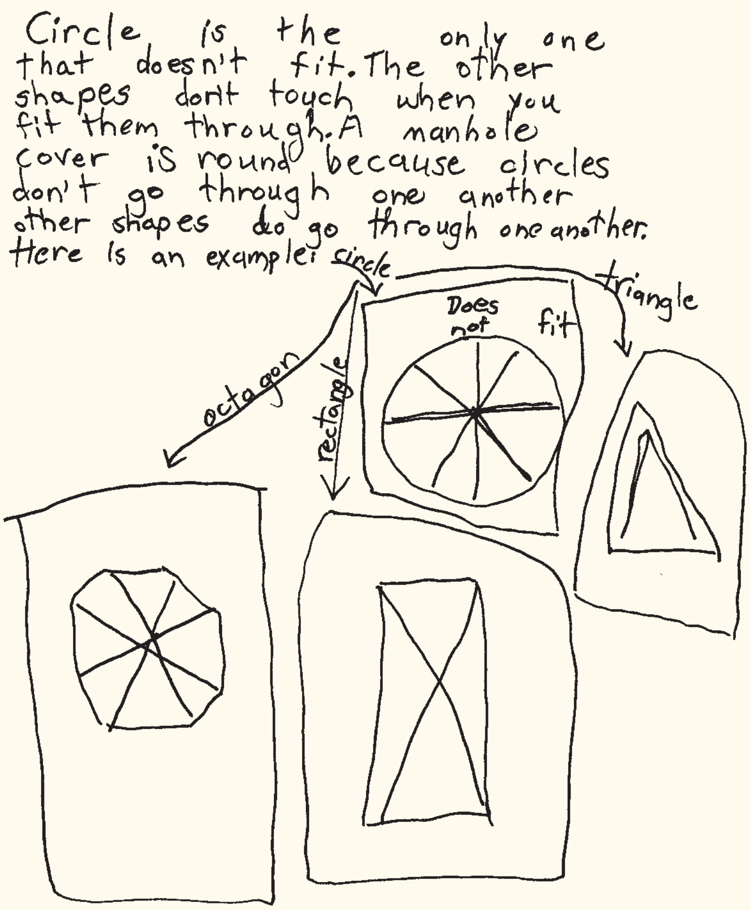


Figure 2

Teresa's response to the Manhole Cover investigation shows that "a manhole cover is round because circles don't go through one another."



are equal, the circle cannot fall in on itself. Such observations underscored for the children how these equal-length diameters of a circle make it uniquely different from other polygons. Teresa described her discoveries in this way: "A circle is the only one that doesn't fit. The other shapes don't touch when you fit them through. A manhole cover is round because circles don't go through one another. Other shapes do go through one another." Teresa's drawings (see **fig. 2**) emphasized the way that different shapes can be turned to slip through the openings. The rectangle and octagon can be tilted on their diagonals, so she drew lines from vertex to vertex on those shapes. Her drawing of the isosceles triangle, however, reflects the twisting motion required to fit that shape through its opening. For the circle, she drew several diameters to signify her repeated attempts and wrote, "Does not fit."

Other students described their findings in other ways. Ashley succinctly concluded, "A circle has no angles or no straight sides, so you can't fit a circle through."

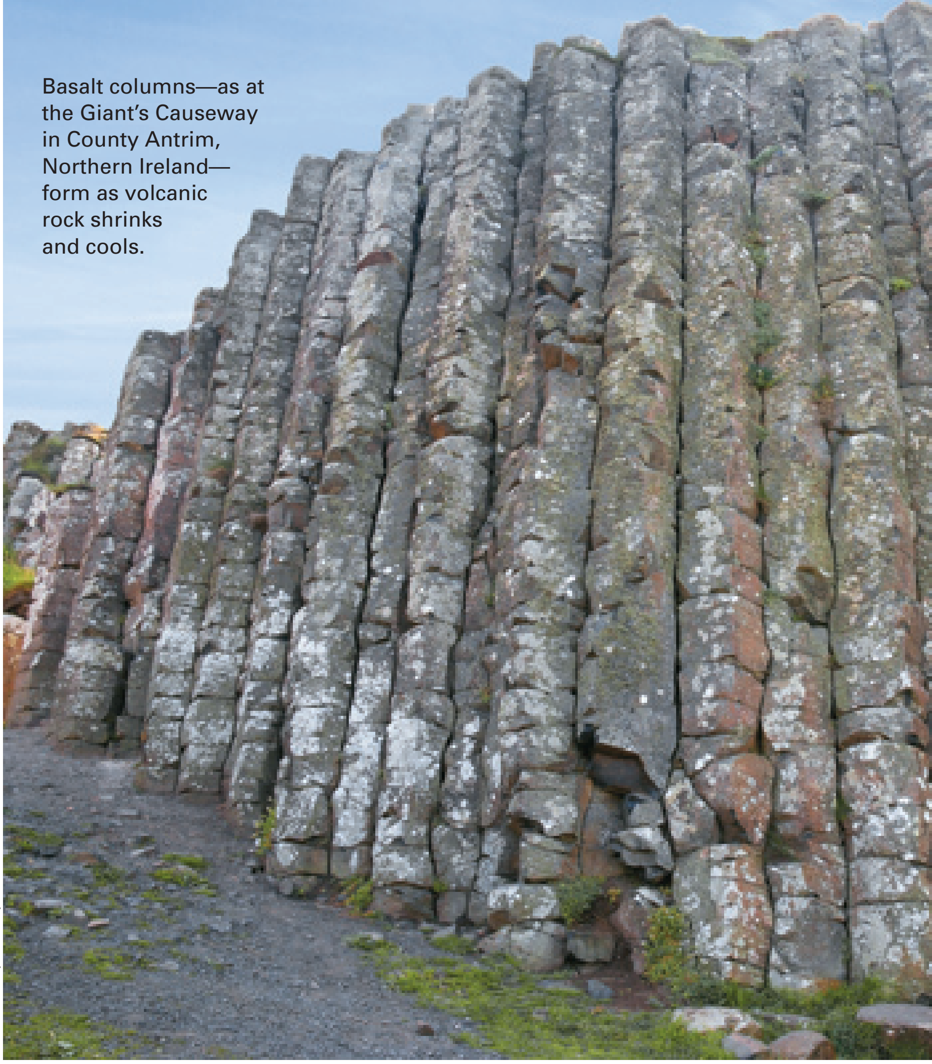
Matthew emphasized the functional application of the investigation: "I think the manhole lid is a circle because if it got sideways, the manhole [lid] wouldn't fall through and hit the person on the head. Most of the time, the other shapes went through [their frame] by angles. A circle has no angles."

After testing these polygons, the children also enjoyed rotating each shape to fit exactly inside its cutout. Such actions give them practice in recognizing the rotational symmetry of each polygon.

The children's descriptions and manipulations of the shapes demonstrated that a polygon's sides are always shorter than its diagonal. (Some exceptions to this observation exist if we consider concave shapes.) However, because a circle does not possess this attribute, it becomes the safest choice, and therefore the most appropriate shape, for a manhole cover.

The children logged all these observations in their mathematics journals. After sharing their findings during a whole-class meeting, students recorded their results on a large chart that summarized their ongoing geometry insights.

This particular activity was only one of several that the teacher planned for her students. We share it here because it highlights the unique relationship between form and function of shapes in the world. To pursue this idea further, we devote the remainder of the article to a range of explorations that demonstrate how inquiring about shape can integrate



Basalt columns—as at the Giant's Causeway in County Antrim, Northern Ireland—form as volcanic rock shrinks and cools.

Jane McIlroy/iStockphoto.com

mathematics and science in interesting ways. The examples reflect the functionality of shape in both constructed and natural contexts. Fostering a spirit of inquiry is an essential habit of mind that builds a classroom of thinking, conjecturing students. Teachers can engender this attitude by demonstrating the reasons for shapes through various activities that set the tone for discovering the "why" behind shapes and designs. Such activities can spark children's interest in posing this question themselves in other contexts throughout the curriculum. Just as the manhole cover example tied to the study of polygons, the following activities incorporate such concepts and skills as tessellations, angle measurement, radial and bilateral symmetry, and ratio.

Shapes in Nature: Efficiency and Survival

In nature, shape is generally determined by the principle of efficiency: taking the shortest path, conserving energy, or making the tightest fit (Murphy 1993). For example, soap bubbles are spherical because trapped air exerts pressure equally in all directions on the soap-and-water mixture, causing it to expand. The film resists being stretched, so it is energy-efficient to assume a spherical shape, which has the least amount of surface area for a given volume (Wick 1997). Children can inquire into this phenomenon by dipping polygon-shaped wires into a soap solution. The film assumes the polygonal shape as it adheres to the wire, but if blown gently and released from the frame, the film reshapes into a sphere.

Entomology

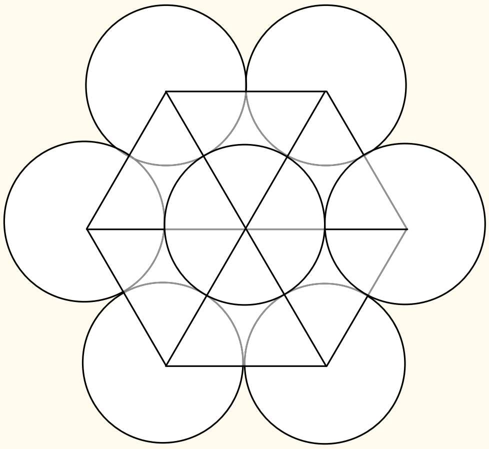
Investigating why honeycomb cells are hexagonal can build children's understanding of the attributes of a hexagon and its relationship to a circle. Lots of energy is required for bees to make wax. When freshly made, each cell of wax is cylindrical; its rounded shape uses the least amount of wax to form the greatest possible area. As the cells align with one another, the weight of the wax pressing equally on all sides compresses the center cells into hexagons (see **fig. 3**). Teachers can challenge children to create this transformation themselves: Have them make a short cylinder out of clay, and then ask, "What shape is the end of that cylinder? How can you reshape the end so it is a regular hexagon and not a circle?" Once they have accomplished this task, invite them to create the hexagon in another way: "Now let's see what happens when we place lots of cylinder pieces next to one another." Have the children make a clay cylinder about the size of a pencil's diameter and then cut it into seven pieces, each about one-half inch long. Stand the cylinders on their ends, surrounding one central cylinder with the other six. Next, roll the entire bundle, pushing evenly but forcefully. Ask the children what they notice. The pressure of the surrounding cylinders will reshape the center one into a hexagon. This simple experiment demonstrates how bees conserve

Figure 3

The weight of fresh wax pressing equally on all sides of cylindrical honeycomb cells compresses the center cells into hexagons.

Figure 4

Using tracings of pennies, students can demonstrate why honeycomb cells are regular hexagons.



energy by using the least possible amount of wax (a circle encloses the greatest amount of area with a fixed amount of material), thus increasing their likelihood of survival. Pressing the circles together is what remolds the wax into hexagons. Children can measure the 120-degree angles formed at the meeting points and observe that three of these angles equal a complete rotation of 360 degrees. Younger children can build honeycomb models with pattern blocks and discuss how the cells fit together with no spaces (tessellate), thus conserving space and making the hive strong.

A related activity shows children why the honeycomb cells are hexagons. Trace around a penny, and then surround that circle with six other penny-sized circles. Each outer penny touches the inner one at only one point, so there are six tangential points at equal intervals around the central penny. Mark the center of each circle and draw lines to connect the centers of adjacent pennies in the outer ring. The children should notice that the result is a regular hexagon. The centers of the outer circles can also be connected to the center of the central penny, thus dividing the hexagon into six congruent equilateral triangles (see **fig. 4**). This demonstration preserves the original circular shapes so to better compare them to the hexagon. Children can defend why the angles in the triangles are each 60 degrees and why the triangles will always be equilateral, even when other circular objects are traced. Such discussions highlight that equal-sided polygons have equal angles. Children might explore this relationship further by using different-sized circles. Their discoveries can help show that all circles are similar, or proportionate.

Botany

Kernels of corn are a good example of the principle of efficiency in botany (Murphy 1993). Corn kernels grow in staggered, interlocking rows, thus conserving energy by being tightly packed. Children can examine ears of corn to find that a kernel in one row touches two kernels in the row below. The junctures form angles of approximately 120 degrees. Looking at the single kernel as a center, children will see that six kernels surround it (two below, two above, and one on either side), an arrangement similar to the penny demonstration. Such hexagonal packing can also be seen in the commercial world, described later in this article.

Earth Science

Hexagonal formations can be found in earth science as well. Columnar basalt, an igneous rock, shrinks and cracks as it cools. The cracks form angles close to 120 degrees, creating hexagonal patterns. Devil's Postpile National Monument in California is a stunning example of this kind of formation (see **fig. 5** and the phtotogallery on the NPS 2006 Web site).

Rocks (and cement) also crack under circumstances other than cooling, and the resulting shapes show different patterns. Rocks begin to crack when they are under pressure, perhaps as a result of water seeping into tiny crevices and then freezing and expanding. Once begun, the crack follows the path

Figure 5

Columnar basalt at Devil's Postpile National Monument shrank and cracked simultaneously as it cooled, forming hexagonal patterns.

of least resistance, which is generally a straight line (Murphy 1993). Now the greatest pressure on the rock on either side of the crack is at the midpoint of that line. Over time, a second crack will begin at that point. Because equal pressure is exerted on each side of the midpoint, the new crack forms perpendicular to the first crack. Perpendicular fissures in rocks, then, are the result of sequential cracking, rather than the simultaneous cooling of basalt.

Children can observe these differing mathematical cracking patterns to help explain geological histories. They might explore and photograph examples of rock, pavement, or drying mud in their own schoolyard or community or find images on the Internet (see **fig. 6**). Exploring the cracking and splitting apart of surfaces can prompt children to pose larger questions about the weathering effects of erosion: Where else do we see rocks cracking? How do these cracks occur? Why are some mountain ranges different shapes and sizes? How did the shape of steppes and buttes come to be? Introductory activities promote this inquisitive spirit.

Figure 6

Rocks with approximately perpendicular cracks are the result of sequential cracking.

Biology

In the field of life science, adaptation for survival is another key concept. How symmetry relates to survival comes into play when children investigate the shapes of creatures, particularly with regard to their appendages. Teachers can read aloud *One Is a Snail, Ten Is a Crab* (Sayre and Sayre 2003), which tells the story of different creatures' legs added together to show the numbers one through ten and the multiples of ten to one hundred. The book often prompts a discussion about the number of legs found on different creatures. Teachers can invite children to make a list of their findings (see **table 1**), which may cause children to wonder why no creatures correlate to many of the odd numbers. Teachers can encourage their students to offer some hypotheses to explain this phenomenon. Related discussions highlight the need for creatures to balance their weight and to move about in an efficient manner. These observations can lead to a discussion about the body symmetry of humans and many other animals.

Teachers can then ask children what they notice about the symmetry of trees and plants. Children will notice that these shapes have many lines of symmetry. Continue to challenge them by asking, "What benefit is it for plants to have this radial symmetry?" Students' observations should highlight the fact that trees and plants lack locomotion and must remain fixed in one spot. To reach light, water, and nutrients, they send out roots and branches in all directions from their centers. So, radial symmetry serves the survival needs of trees and flowers. In every case, creatures' and plants' shapes are tied to their mobility and need for survival (Ross 1996).

Constructed Shapes

From bridges to coat hangers, from silos to umbrellas—humans employ geometric principles in design. Looking at everyday objects and posing the question, "Why are things shaped as they are?" can engender an inquisitive stance about the most mundane objects. For instance, why is a coat hanger a triangle shape and not square, rectangular, or hexagonal? Furthermore, why is it an isosceles and not a right or a scalene triangle? Such questions invite children to think about how an object's shape is determined by its function. An isosceles triangle best mirrors the shape of a human's upper torso.

Table 1

Number of Legs on Creatures

1	2	3	4	5	6	7	8	9	10
worm	person		dog cat	starfish	insect		octopus spider		crab

The angles from the triangle's apex approximate the slope of our shoulders, and the two equal sides match the bilateral symmetry of our bodies.

Another interesting application of circles from the commercial world involves packing. Ask students to imagine that they have to pack some very expensive, fragile glasses into a crate. Invite them to arrange a set of discs into a flat box of any shape (Mold 1967). An interesting conversation can occur around two possible arrangements. To prevent the glasses from breaking, where must students place some stiff cardboard separators? Have them draw circles to represent the glasses and lines between the glasses to show where to place the cardboard. The lines correspond to those places where the glasses are tangent. **Figure 7a** shows a square pattern of cardboard inserts, and **figure 7b** illustrates a hexagonal pattern for packing (as illustrated previously by the kernels of corn). Students could examine each packing solution and determine which way uses less space. Clearly, the hexagonal packing conserves the most space because the circles can be packed together more tightly.

Have the children discuss why square packing is the better way to ship fragile objects (fewer points of contact), whereas hexagonal packing is more useful and efficient when no separators are needed (Benenson and Neujahr 2002). Insights about shape arrangement can give children a new perspective for viewing the world and can inspire them to do further research. Can they find other examples of such packing? For instance, hexagonal packing examples might include timber on a logging truck (see **fig. 8**), straws in a box, firewood in a woodpile, or marbles in a jar. Examples of square packing could include boxes of chalk or light bulbs as well as egg cartons.

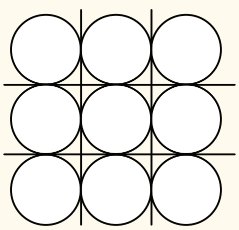
Another widespread use of a constructed shape is the *golden rectangle*. Thought to be the most eye-pleasing rectangle because of its ratio of length to width (approximately 1.618), it looks neither too thin nor too wide. Students can hunt for golden rectangles in their school and home environments. Ask them to first predict if a particular rectangle is golden, then measure the dimensions, and finally divide the length by the width to see if they obtain the ratio of 1.6. They will find that many picture frames are golden rectangles, such as a three-by-five-inch and a five-by-eight-inch (5 ÷ 3 = 1.67, and 8 ÷ 5 = 1.6). Teachers might share examples from the art world, such as the Parthenon in Greece and some of Piet Mondrian's abstract works (Queen's University 2006).

Parts of the human body are said to reflect this golden ratio, too. One example is a person's overall height compared to the distance from the floor to the person's navel. Dividing the distance to the navel into one's total height obtains the ratio of approximately 1.6. Observe, too, how companies use this well-proportioned rectangle in their packaging. Credit cards and some food packaging reflect this ratio. One of the more interesting locations for this rectangle is on the faces of analog clocks and watches (Garland 1987). When these time pieces are advertised, they are usually set at 8:20 or 10:10, suggesting that these times are the most visually appealing because our eyes tend to construct a golden rectangle by using the hands of the clock as partial diagonals (see **fig. 9**). Invite students to find examples of watches on display and then create such rectangles to see if their findings confirm this idea. Such investigation helps children see how shapes with this special ratio are purposely designed for commercial reasons.

Figure 7

Packing drinking glasses shows an interesting application of circles from the commercial world.

(a) Square packing is a safer solution for fragile round objects.



(b) Hexagonal packing, however, uses less space.

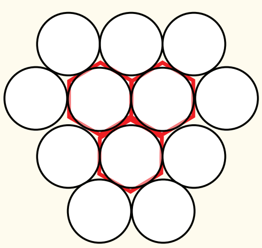


Figure 8

Logs on a truck are another example of hexagonal packing.

Fourth graders exploring natural and manufactured shapes in a real-world context determined that—geometrically—circles are uniquely suited to be manhole covers .

Reasons for shapes also abound in the field of architecture. Children can gain perspective about the shape of a house by engaging in an experiment that highlights the relationship between perimeter and area (Whitin 1993; Zaslavsky 1989). Set the challenge by saying, "Pretend you live in a village in which you have limited resources, and you must make almost everything yourself. Let's pretend that this piece of string (32 cm in length) represents the amount of material you have available to build a house. How can you use this string to encompass the greatest possible area?" The children can outline their shapes on a piece of centimeter paper and then count the squares to compute the area. They will notice that the area becomes the greatest as the shape begins to approximate a circle. This important relationship can lead them to further inquire about constructing and using round homes or buildings (Grifalconi 1986). Some examples include igloos, tepees, yurts, and sports arenas.

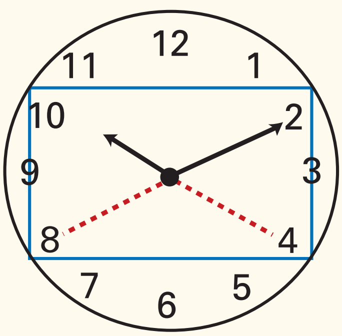
The relationship of perimeter to area can be extended into three dimensions as well. Spheres yield the greatest possible volume for a fixed amount of surface area (as the soap bubbles illustrated earlier). Given this relationship, why aren't soup containers spherical? Even children realize how cumbersome it would be to have spheres rolling around the kitchen shelves! However, the next best shape is a cylinder, which yields the greatest volume among prisms with the same base perimeter. To test this idea, children can bend a piece of cardstock into a cylinder and fill it with rice or sand and weigh the amount as a measure of its volume. They can fold another piece of cardstock of the same size into a rectangular solid or a prism and compute those volumes as well.

Once children have proven that the cylinder shape yields the greatest volume for a given surface area, they can look for everyday applications of this principle. For instance, various canned goods are packaged in cylindrical containers. Children can also draw on their knowledge of circles to justify the shape of other examples. Because a circle is defined as a set of points equidistant from a center point, children can reason that a cylinder is a shape that is least likely to burst because the pressure on its contents is equally distributed. This same principle explains why the hatches on a submarine are circular (Gates 1995). Silos are another example; more grain can be stored in this shape, and the continuously curved side allows the grain to fill more completely, minimizing the potential for a pocket of air to develop and allow bacteria to grow.

These examples demonstrate how objects from nature and manufacturing can provide interesting cross-disciplinary investigations. **Table 2** summarizes some main mathematical content embedded in these various real-world applications.

Figure 9

The hands of a clock set at 10:10 or 8:20 suggest a golden rectangle.



Why Explore?

The activities that we describe suggest many benefits to understanding the reasons for shapes:

- Subject fields become integrated. The examples cited in this article demonstrate that children learn not only important mathematical principles during such investigations, such as symmetry, angle measurement, ratio, and area, but also foundational concepts in other disciplines. For instance, children learn such scientific concepts as adaptation and economic principles of cost-benefit considerations involved with packaging goods and designing products. Interdisciplinary investigations allow children to make better connections in their learning.
- Learners become more critical consumers. When students realize that corporate advertising personnel use shape to influence a buyer's purchasing decisions, they become more astute in analyzing the decisions they make as consumers. For instance, which minivan really has the most usable storage space? Which shapes in the grocery store give the illusion of holding more? (Executives are reluctant to change product prices but will often alter the shape, size, and volume of a company's container instead.)
- Students appreciate and understand geometric elements of design in everyday manufactured objects. There are reasons why manhole covers are round and coat hangers are isosceles triangles. Examining the reasons for shapes sparks a curiosity about the commonplace and fosters a spirit of reflection and inquiry about our natural environment. Why are beach pebbles smooth? Why are bird eggs ovoid? Vivian Gornick's words echo this inquisitive stance toward the world:

> The natural biologist walks through a city park, across a suburban lawn, past an open shopping mall and is half-consciously wondering: Why two leaves instead of three? Why pink flowers instead of white? Why does the plant turn this way instead of that way? Such ruminations go on without end in the scientist's mind, a continuous accompaniment to the rhythm of daily life. (1983, pp. 38–39)

In summary, the question, "Why are things shaped the way they are?" provides an exciting, rewarding avenue to explore how mathematical concepts intersect with the work of scientists, engineers, artists, and others in the world beyond the classroom.

Table 2

Mathematical Content of Real-World Shapes

Real-World Applications	Mathematical Content
Manhole covers, umbrellas, submarine doors	Attributes of a circle
Soap bubbles, soup cans	Surface area, volume
Honeycombs	Tangents, similarity, angles, tessellation
Cracking	Angles; horizontal, vertical, and perpendicular lines
Body and plant shapes	Bilateral and radial symmetry
Coat hangers	Bilateral symmetry, angles
Packaging	Tangents, area
Golden rectangle	Ratio
Architecture	Perimeter, area, polygons

References

Benenson, Gary, and James L. Neujahr. *Packaging & Other Structures*. Portsmouth, NH: Heinemann, 2002.

Garland, Trudi Hammel. *Fascinating Fibonaccis: Mystery and Magic in Numbers*. Palo Alto, CA: Dale Seymour Publications, 1987.

Gornick, Vivian. *Women in Science: Portraits of a World in Transition*. New York: Simon and Schuster, 1983.

Mold, Josephine. *Circles*. London: Cambridge University Press, 1967.

Murphy, Pat. *By Nature's Design*. San Francisco: Chronicle Books, 1993.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000.

———. "Figure This! Why Are Most Manhole Covers Round?" 2004. www.figurethis.org/challenges/c04/challenge.htm#hint.

National Park Service (NPS), U.S. Department of the Interior. "Devil's Postpile National Monument, California." 2006. www.nps.gov/depo.

Queens University, Kingston, Ontario, Canada. "Math and Art: The Golden Rectangle." 2006. educ.queensu.ca/~fmc/october2001/GoldenArt.htm.

Whitin, David J. "Looking at the World from a Mathematical Perspective." *Arithmetic Teacher* (April 1993): 438–41.

Zaslavsky, Claudia. "People Who Live in Round Houses." *Arithmetic Teacher* 37 (September 1989): 18–21.

Children's Literature

Gates, Phil. *Nature Got There First: Inventions Inspired by Nature*. New York: Kingfisher, 1995.

Grifalconi, Ann. *The Village of Round and Square Houses*. Boston: Little Brown, 1986.

Ross, Catherine Sheldrick. *Squares*. Toronto: Kids Can Press, 1996.

Sayre, April Pulley, and Jeff Sayre. *One Is a Snail, Ten Is a Crab*. London: Walker Books Ltd., 2003.

Wick, Walter. *A Drop of Water*. New York: Scholastic, 1997. ▲